MODELING, SIMULATION AND STABILITY ANALYSIS OF A GEORGE CONSTANTINESCO TORQUE CONVERTER

Ion ION-GUTA*
University of Pitesti, Romania

Article history:
Received: 20.01.2016; Accepted: 18.04.2016.

Abstract. *This paper is a dynamic analysis of one of the many variants of mechanical torque converter designed by the eminent romanian inventor George Constantinesco. The analyzed model is composed of nine-bar linkage that transmit motion from the primary motor to the output shaft through one-way clutches. In the first stage we are determined the equations of motion in matrix form and by their numerical solution we are indicate the main features of the torque converter. Analysis of dynamic stability of mechanical transmission is made through the linear analysis with Bode diagrams. Simulation of transmission operation is performed with AMESim.*

Keywords: nine-bar linkage, closed loop, two degrees of freedom system, simulation, Lagrange's equations

INTRODUCTION

The analyzed mechanism is a planar nine-bar linkage arranged in three closed loops, shown in Fig. 1. In Fig. 2, shows the simulation model of torque converter developed by AMESim program. It is a two degree of freedom mechanism, the degrees being the crank angle φ_1 and the angular displacements of the links 6 and 8, φ_6 and φ_8 respectively. The bar system 5, 6, 7 and 8with the one-way clutches 9 and 10, form a unidirectional mechanism that rotates the output shaft 11 in a certain sense. The imposed external forces are the driving torque T_1, provided by a drive motor at constant speed and the loading torques T_6, T_8 acting on the bars 6, 8 respectively.

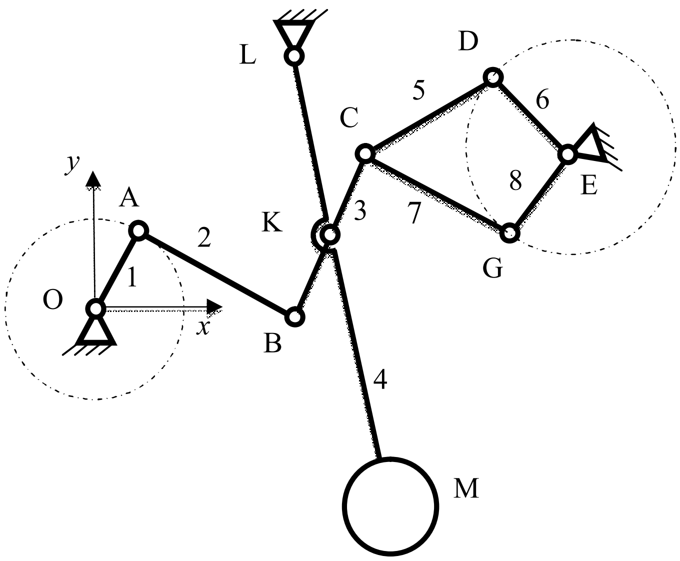

Figure 1. The G. Constantinesco's torque converter diagram.

* Corresponding author. Email: ion.ion@upit.ro

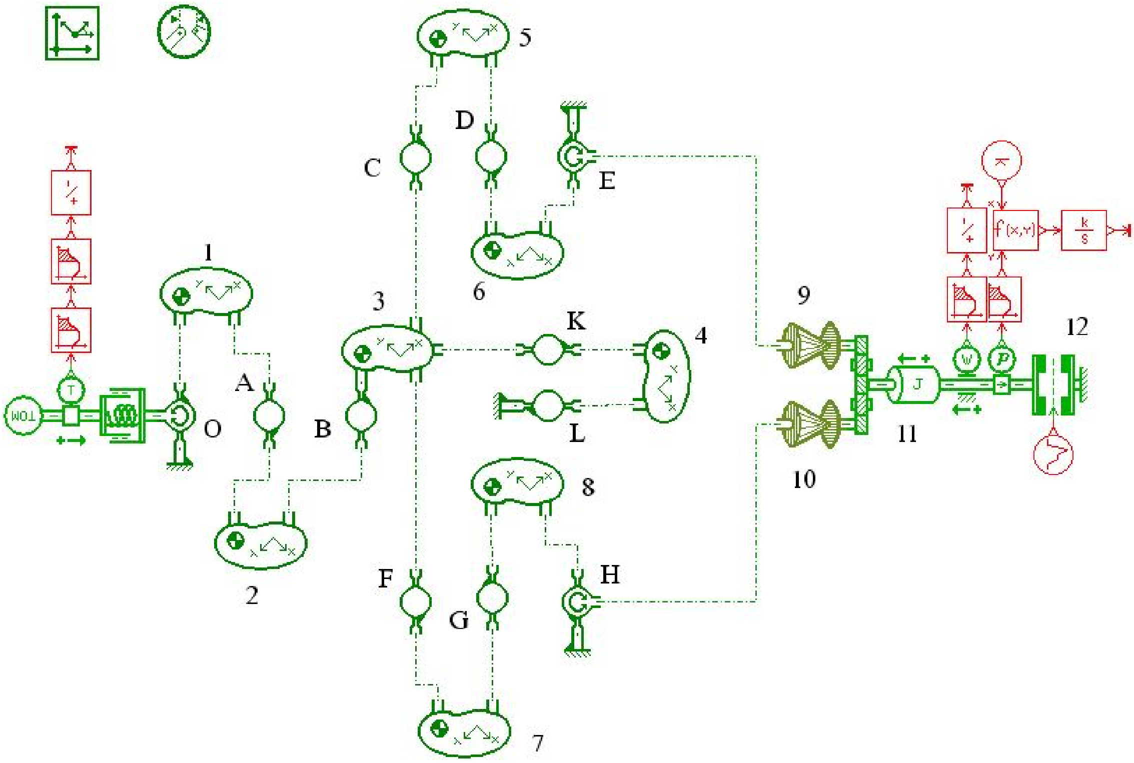

Figure 2. Simulation scheme of torque converter modeled by AMESim

EQUATIONS OF MOTION

In relation to the *oxy* reference system, the coordinates of the joints A, B, C, K, D, G and center of mass M are given by relations:

$$x_A = l_1 \cos\varphi_1 \tag{1}$$

$$y_A = l_1 \sin\varphi_1 \tag{2}$$

$$x_B = x_A + l_2 \cos\varphi_2 \tag{3}$$

$$y_B = y_A + l_2 \sin\varphi_2 \tag{4}$$

$$x_K = x_B + l_{3K} \cos\varphi_3 = x_L - l_{4K} \cos\varphi_4 \tag{5}$$

$$y_K = y_B + l_{3K} \sin\varphi_3 = y_L - l_{4K} \sin\varphi_4 \tag{6}$$

$$x_M = x_L - l_{4M} \cos\varphi_3 \tag{7}$$

$$y_M = y_L - l_{4M} \sin\varphi_3 \tag{8}$$

$$x_C = x_B + l_{3C} \cos\varphi_3 \tag{9}$$

$$y_C = y_B + l_{3C} \sin\varphi_3 \tag{10}$$

$$x_D = x_C + l_5 \cos\varphi_5 = x_E + l_6 \cos\varphi_6 \tag{11}$$

$$y_D = y_C + l_5 \sin\varphi_5 = y_E + l_6 \sin\varphi_6 \tag{12}$$

$$x_G = x_C + l_7 \cos\varphi_7 = x_E + l_8 \cos\varphi_8 \tag{13}$$

$$y_G = y_C + l_7 \sin\varphi_7 = y_E + l_8 \sin\varphi_8 \tag{14}$$

To simplify writing , in what follows will use the following notation

$$\sin\varphi_i = s_i; \cos\varphi_i = c_i \quad (i = 1 \ldots 8) \tag{15}$$

The kinematics may be defined by writing the constraint equations along and perpendicular to the axe *ox*.

$$f_1 = l_1c_1 + l_2c_2 + l_{3K}c_3 + l_{4K}c_4 - x_L = 0 \quad (16)$$
$$f_2 = l_1s_1 + l_2s_2 + l_{3K}s_3 + l_{4K}s_4 - y_L = 0 \quad (17)$$
$$f_3 = l_1c_1 + l_2c_2 + l_{3C}c_3 + l_5c_5 - l_6c_6 - x_E = 0 \quad (18)$$
$$f_4 = l_1s_1 + l_2s_2 + l_{3C}s_3 + l_5s_5 - l_6s_6 - y_E = 0 \quad (19)$$
$$f_5 = l_1c_1 + l_2c_2 + l_{3C}c_3 + l_7c_7 - l_8c_8 - x_E = 0 \quad (20)$$
$$f_6 = l_1s_1 + l_2s_2 + l_{3C}s_3 + l_7s_7 - l_8s_8 - y_E = 0 \quad (21)$$

In adition, in the case of a constant speed input ω_1 of the crank, a further equation

$$f_7 = \varphi_1 - \omega_1 t = 0 \quad (22)$$

is required. The above equations may be represented in suffix notation as

$$f_j(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7, \varphi_8) = 0 \quad (23)$$

where $j = 1 \dots 7$.

The constraint equations expressed in terms of velocities may be be derived from (23) and written in suffix notation as,

$$V_j(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7, \varphi_8, \dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4, \dot{\varphi}_5, \dot{\varphi}_6, \dot{\varphi}_7, \dot{\varphi}_8) = 0 \quad (24)$$

The equations of motion are derived using the Lagrangian multipliers method. This may be stated as follows [1,2]:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} = Q_i + \sum_{j=1}^{7} \lambda_j \frac{\partial f_j}{\partial q_i} \quad (25)$$

where E is the total kinetic energy of the system, q_i and $\dot{q}_i$ are the generalised coordinates and velocities, Q_i the generalised forces including conservative and nonconservative effects, λ_j the Lagrangian multipliers and f_j the constraint equations.

The generalised forces Q_i may be written in the form

$$Q_i = \sum_{j=1}^{7}\left(m_j \vec{g} \cdot \frac{\partial \vec{r}_j}{\partial q_i} + \vec{T}_j \cdot \frac{\partial \vec{\varphi}_j}{\partial q_i}\right) \quad (26)$$

where m_j represents the masses, $\vec{g}$ the gravitational vector, $\vec{r}_j$ the coordinate vector of the point of application of gravitational forces and $\vec{T}_j$ the vector of torques.

Let the generalised coordinates q_i be $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7$ and φ_8. Now in order to obtain the kinetic energy in terms of these coordinates the velocities of the centres of masses must be found. These can be obtained by writting the *x* and *y* coordinates of the centres of masses and taking the first derivative. In this case, for $C_1 = O;\ C_2 = A;\ C_3 = K;\ C_4 = M;\ C_5 = D;\ C_6 = E;\ C_7 = G;\ C_8 = E$ is obtained:

$$v_{C1} = 0 \quad (27)$$
$$v_{C2} = v_A = l_1\dot{\varphi}_1 \quad (28)$$
$$v_{C3} = v_K = l_{4K}\dot{\varphi}_4 \quad (29)$$
$$v_{C4} = v_M = l_{4M}\dot{\varphi}_4 \quad (30)$$
$$v_{C5} = v_D = l_6\dot{\varphi}_6 \quad (31)$$
$$v_{C6} = v_{C8} = 0 \quad (32)$$
$$v_{C7} = v_G = l_8\dot{\varphi}_8 \quad (33)$$

Hence the total kinetic energy of the machanism may be expressed as:

$$E = J_1\frac{\dot{\varphi}_1^2}{2} + J_2\frac{\dot{\varphi}_2^2}{2} + J_3\frac{\dot{\varphi}_3^2}{2} + J_4\frac{\dot{\varphi}_4^2}{2} + J_5\frac{\dot{\varphi}_5^2}{2} + J_6\frac{\dot{\varphi}_6^2}{2} + J_7\frac{\dot{\varphi}_7^2}{2} + J_8\frac{\dot{\varphi}_8^2}{2} + \frac{m_2 l_1^2\dot{\varphi}_1^2}{2} + \frac{m_3 l_{4K}^2\dot{\varphi}_4^2}{2} + \frac{m_4 l_{4M}^2\dot{\varphi}_4^2}{2} + \ + \frac{m_5 l_6^2\dot{\varphi}_6^2}{2} + \frac{m_7 l_8^2\dot{\varphi}_8^2}{2} \tag{34}$$

Based on notations $q_i = \varphi_i;\ \dot{q}_i = \dot{\varphi}_i\quad (i = 1 \dots 8)$, we obtain

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_1}\right) - \frac{\partial E}{\partial q_1} = (J_1 + m_2 l_1^2)\ddot{\varphi}_1 \tag{35}$$

$$\vdots$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_8}\right) - \frac{\partial E}{\partial q_8} = (J_8 + m_7 l_8^2)\ddot{\varphi}_8 \tag{36}$$

The generalized forces Q_j are given by the following relationship

$$Q_1 = T_1 - (m_2 + m_3) g l_1 c_1 \tag{37}$$

$$\vdots$$

$$Q_8 = -m_7 g l_8 c_8 - T_8 \tag{38}$$

The bars 6 and 8 transmit to the output shaft movement through two one-way clutch, so that they are charged under load only when rotating counterclockwise.

$$T_6 = \begin{cases} T_{11}\ if\ (\dot{\varphi}_6 - \dot{\varphi}_8) \geq 0 \\ 0\ if\ (\dot{\varphi}_6 - \dot{\varphi}_8) < 0 \end{cases} \tag{39}$$

$$T_8 = \begin{cases} 0\ if\ (\dot{\varphi}_6 - \dot{\varphi}_8) \geq 0 \\ T_{11}\ if\ (\dot{\varphi}_6 - \dot{\varphi}_8) < 0 \end{cases} \tag{40}$$

where

$$\begin{cases} T_{11} = T_{12} + J_{11}\ddot{\varphi}_{11} \\ \dot{\varphi}_{11} = max(\dot{\varphi}_6, \dot{\varphi}_8) \end{cases} \tag{41}$$

The terms of the type $\sum_{j=1}^{7}\lambda_j\frac{\partial f_j}{\partial q_i}$ are shown in the following relationships:

$$\sum_{j=1}^{7}\lambda_j\frac{\partial f_j}{\partial q_1} = -\lambda_1 l_1 s_1 + \lambda_2 l_1 c_1 - \lambda_3 l_1 s_1 + \lambda_4 l_1 c_1 - \lambda_5 l_1 s_1 + \lambda_6 l_1 c_1 + \lambda_7 \tag{42}$$

$$\vdots$$

$$\sum_{j=1}^{7}\lambda_j\frac{\partial f_j}{\partial q_8} = \lambda_5 l_8 s_8 - \lambda_6 l_8 c_8 \tag{43}$$

The equations of motion, determined by Lagrange multipliers can be written as:

$$0 = T_1 - (m_2 + m_3) g l_1 c_1 + \lambda_1 l_1 s_1 - \lambda_2 l_1 c_1 + \lambda_3 l_1 s_1 - \lambda_4 l_1 c_1 + \lambda_5 l_1 s_1 - \lambda_6 l_1 c_1 - \lambda_7 \tag{44}$$

$$J_2\ddot{\varphi}_2 = -m_3 g l_2 c_2 + \lambda_1 l_2 s_2 - \lambda_2 l_2 c_2 + \lambda_3 l_2 s_2 - \lambda_4 l_2 c_2 + \lambda_5 l_2 s_2 - \lambda_6 l_2 c_2 \tag{45}$$

$$J_3\ddot{\varphi}_3 = -m_3 g l_{3K} c_3 + \lambda_1 l_{3K} s_3 - \lambda_2 l_{3K} c_3 + \lambda_3 l_{3C} s_3 - \lambda_4 l_{3C} c_3 + \lambda_5 l_{3C} s_3 - \lambda_6 l_{3C} c_3 \tag{46}$$

$$(J_4 + m_3 l_{4K}^2 + m_4 l_{4M}^2)\ddot{\varphi}_4 = m_4 g l_{4M} c_4 + \lambda_1 l_{4K} s_4 - \lambda_2 l_{4K} c_4 \tag{47}$$

$$J_5\ddot{\varphi}_5 = \lambda_3 l_5 s_5 - \lambda_4 l_5 c_5 \tag{48}$$

$$(J_6 + m_5 l_6^2)\ddot{\varphi}_6 = -m_5 g l_6 c_6 - T_6 - \lambda_3 l_6 s_6 + \lambda_4 l_6 c_6 \tag{49}$$

$$J_7\ddot{\varphi}_7 = \lambda_5 l_7 s_7 - \lambda_6 l_7 c_7 \tag{50}$$

$$(J_8 + m_7 l_8^2)\ddot{\varphi}_8 = -m_7 g l_8 c_8 - T_8 - \lambda_5 l_8 s_8 + \lambda_6 l_8 c_8 \tag{51}$$

Equations (44) through to (51) are second order non-linear differential equations. Together with the following six equations which are obtained by taking the second derivatives of constraint equations (16) through to (21) they may be solved for $\ddot{\varphi}_2, \ddot{\varphi}_3, \ddot{\varphi}_4, \ddot{\varphi}_5, \ddot{\varphi}_6, \ddot{\varphi}_7, \ddot{\varphi}_8, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ and λ_7.

$$\frac{d^2 f_j}{dt^2} = 0 \quad (j = 1 \dots 6) \tag{52}$$

With notations

$$V = [\ddot{\varphi}_2 \quad \ddot{\varphi}_3 \quad \ddot{\varphi}_4 \quad \ddot{\varphi}_5 \quad \ddot{\varphi}_6 \quad \ddot{\varphi}_7 \quad \ddot{\varphi}_8 \quad \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \lambda_7]^T \tag{53}$$

the equations (44) ... (52) may be written in the following matrix form

$$AV = B \tag{54}$$

If A is a non singular matrix, from the above equation is obtained

$$V = A^{-1}B \tag{55}$$

where

$$\mathrm{V}_j = A_{i,j}^{-1} B_i \quad (i = 1 \dots 14;\ j = 1 \dots 14) \tag{56}$$

or, more expressive

$$V_j = C_{ji} B_i \tag{57}$$

From the above equations may be obtained the following relationships

$$\begin{cases} \ddot{\varphi}_{m+1} = C_{m,i} B_i \\ \lambda_m = C_{m+7,i} B_i \end{cases} \quad (m = 1 \dots 7) \tag{58}$$

from where we can easily calculate the terms λ_m.
To calculate the angular positions and velocities of the bars, we can use the following notation $\varphi_2 = y_1;\ \dot{\varphi}_2 = y_2;\ \varphi_3 = y_3;\ \dot{\varphi}_3 = y_4;\ \varphi_4 = y_5;\ \dot{\varphi}_4 = y_6;\ \varphi_5 = y_7;\ \dot{\varphi}_5 = y_8;\ \varphi_6 = y_9;\ \dot{\varphi}_6 = y_{10};$
$\varphi_7 = y_{11};\ \dot{\varphi}_7 = y_{12};\ \varphi_8 = y_{13};\ \dot{\varphi}_8 = y_{14}$. The equations of motion derived earlier can be expressed in a form equivalent to

$$\begin{cases} \dot{y}_1 = g_1(t, y_1, y_2, \dots, y_{14}) \\ \vdots \\ \dot{y}_{14} = g_{14}(t, y_1, y_2, \dots, y_{14}) \end{cases} \tag{59}$$

which can be easily solved using Runge - Kutta method. The Runge-Kutta method are one step method, i.e. to evaluate y at time $t + \delta t$ we only require the information available at point y at time t. Secondly, they do not require the evaluation of the derivative of the function to be integrated.
Due to the high nonlinearity of the equations of motion, in case to drive with constant speed drive, both engine torque and output shaft speed will oscillate around some average values that can be determined with relations:

$$\tilde{T}_1 = mean(T_1) \tag{60}$$
$$\widetilde{\omega}_{11} = mean(\omega_{11}) \tag{61}$$

CASE STUDY EXAMPLE

For the simulation diagram indicated in Fig. 2 is considered the next practice application with the main database entry as follows [3]: $l_1 = 0.1m$; $l_2 = 0.3m$; $l_{3K} = 0.2m$; $l_{3C} = 0.3m$; $l_{4C} = 0.6m$; $l_{4M} = 0.9m$; $l_5 = l_7 = 0.433m$; $l_6 = l_8 = 0.25m$; $m_1 = 0.1kg$; $m_2 = m_3 = 0.3kg$; $m_4 = 4kg$; $m_5 = m_7 = 0.4kg$; $m_6 = m_8 = 0.3kg$; $J_1 = J_2 = J_3 = J_5 = J_6 = J_7 = J_8 = 0.001kg \cdot m^2$; $J_4 = 2kg \cdot m^2$; $J_{11} = 4kg \cdot m^2$; $\omega_1 = 1500\, rev/min$.
For a variation in time of load torque T_{12} imposed by Fig. 3 are obtained the main characteristics of mechanical torque converter shown in Fig. 4.

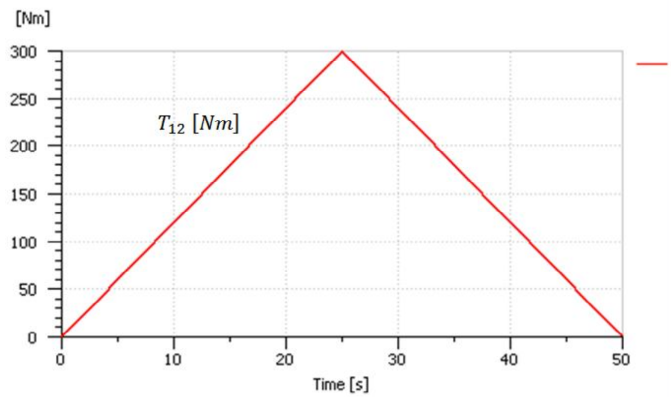

Figure 3. Time variation of the output shaft torque T_{12}

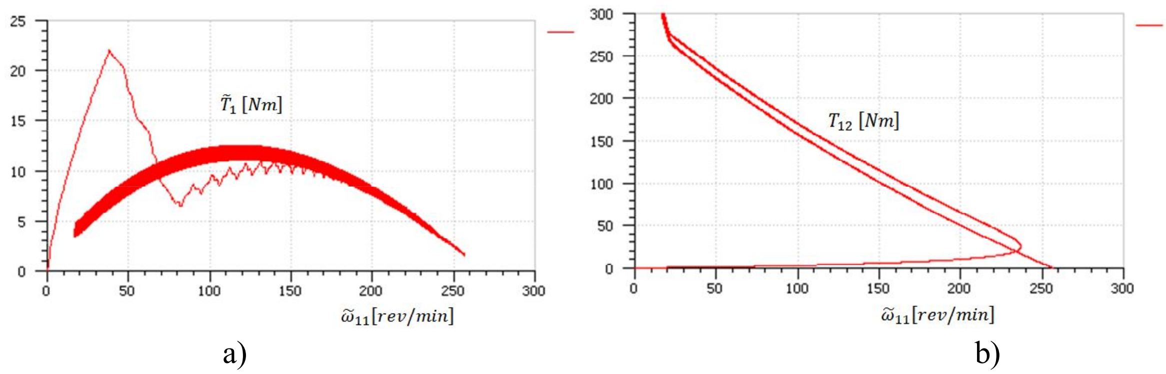

Figure 4. Torque diagrams $\tilde{T}_1$ and T_{12}, depending on the angular velocity of the output shaft

STABILITY ANALYSIS

The steady-state sinusoidal frequency-response of a mechanical system is described by the phasor transfer function $H(j\omega)$. A *Bode plot* is a graph of the magnitude (in dB) or phase of the transfer function versus frequency [4].
Bode plot is one of the most commonly used tools for frequency response. This procedure provides relative stability in terms of gain margin and phase margin. Without determining the analytical expression of the transfer function of the dynamic system studied, in the following we will achieve stability analysis system with AMESim by Linear Analysis tools. In this case is selected as control variable the sine wave output signal ω_1 and as observer the angular

speed of inertial body 4, $\dot{\varphi}_4$. Linearization time is selected at 50 s. The Bode diagram presented as a semi-logarithmic strip plot is shown in Fig. 5.

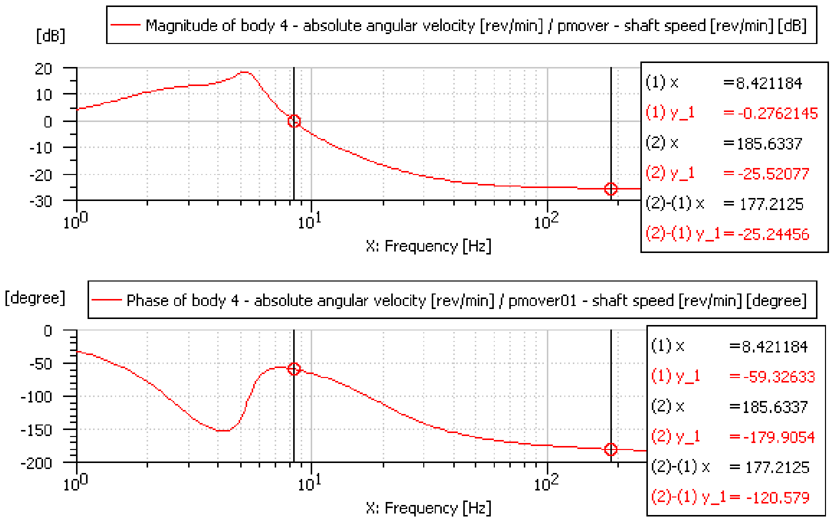

Figure 5. Bode diagrams

CONCLUSIONS

As can be seen from Fig. 4a, the maximum power transmitted through the torque converter is located in the average angular velocity of the output shaft. The maximum power transmission remains approximately constant over a fairly wide range of variation of the angular velocities of the output shaft. The load torque T_{12} has a quasi-linear variation inversely proportional to the output shaft speed, as shown in Fig. 4b. As we can see in the figure above, phase margin is $PM = -59.32633^0 - (-180\ [^0]) = 120.67367[^0]$ and gain margin is $MG = 25.52077\ [db]$. As a result, the system is stable for time $t = 50[s]$.

REFERENCES

[1] J.C. de Jalón, E. Bayo, Kinematic and Dynamic Simulation of Multibody Systems: The Real-Time challenge, Springer-Verlag, New-York, 1994.

[2] Karnopp D.C., D.L. Margolis & R.C. Rosenberg, Systems Dynamics: Modeling and Simulation of Mechatronics Systems, 3rd Edition, 2000, John Wiley &Sons, Inc.

[3] Mechanical Library user's guide, LMS Imagine Lab, 2013.

[4] Information on http://www.plm.automation.siemens.com/en_us/products/lms/imagine-lab/amesim/