SOLUTIONS TO IMPROVE RELIABILITY MODELS USED IN WARRANTY PERIOD

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Abstract: *Determination of reliability based on data obtained during the warranty period of a product requires the application of specific models used for truncated tests. For modeling the reliability, there are used specifically designed computing programs, two situations being possible: complete tests and incomplete tests. however, it is found that in the cases of incomplete tests it is not made distinguish between the censored type testing (which ends when a preset number of products of considered batch failed) and the truncated type testing (which ends at a predetermined time moment). in the case of the incomplete type testing, there is not taken into consideration the time interval between the moment of the last failure and the moment of the end of the experiment (the case of truncated type testing). Therefore, based on the realized study, there is proposed a computing algorithm for modeling the reliability through the usual mathematical laws (uniform, Weibull, exponential, normal) when trying truncated type. The results obtained confirm the usefulness of theoretical and practical computational algorithms proposed.*
For others mathematical models used in truncated tests, must be made a proper calculus algorithm.

Keywords: reliability modeling, computing program, censored tests, truncated tests, uniform law, exponential law, normal law, Weibull law.

PROBLEM FORMULATION

Determination of reliability based on data obtained during the warranty period of a product requires the application of specific models used for truncated tests. For modeling the reliability, there are used specifically designed computing programs [14]. These computer programs make possible the determination of the reliability indicators for the various mathematical models: uniform law, Exp-1P (single-parametric exponential model), Exp-2P (two-parametric exponential model), Normal, Lognormal, Weibull-2P (two-parametric model), Weibull-3P (three-parameteric model), Gamma, G-Gamma (Gamma geenralized), Logistic, Loglogistic, Gumbel.
But working with these computing programs, it was found, however, that these have some limitations in distinguishing between different types of tests. Thus, next there are presented the following research.
For this purpose, will be processed the failure times for 10 homogen products in the framework of a complete test (the test stops after the failure of all components) – presented in Table 1.

Table 1. The values for failure times.

No.	*Failure time [hours]*
1	102
2	110
3	124
4	140
5	152
6	168

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No.	Failure time [hours]
7	180
8	186
9	206
10	220

The computing program analyses the available mathematical models, depends on the correlation rank for the statistical distribution (the number of failed elements is F = 10, and the number of the supervised elements that were not damaged is S = 0, considered suspended dates), presented in hierarchical order (figure 1). Note that the most likely model is the Weibull 3-P model (β = 2,3745; γ = 110,0961 ore; η = 62,2000 ore), with a correlation rank ρ = 0,9906 (fig. 2).

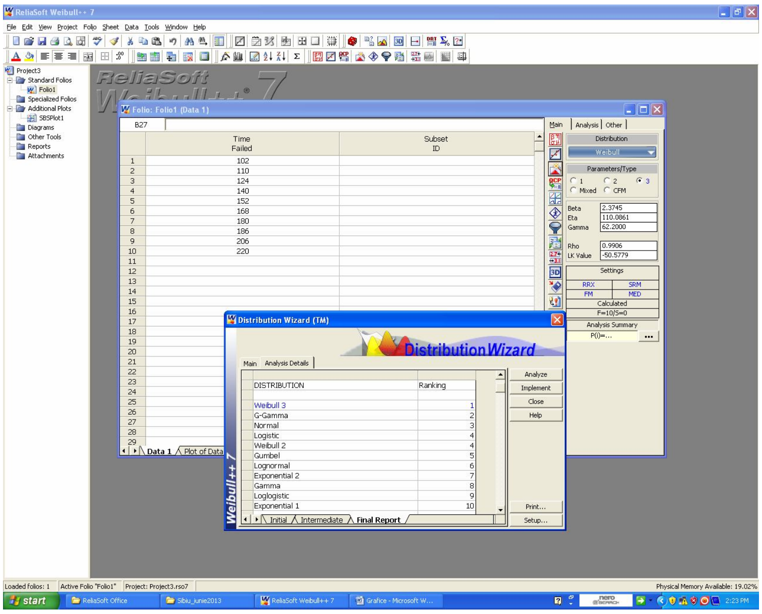

Figure 1. Comparative analysis of complete patterns obtained for testing (F = 10, S = 0).

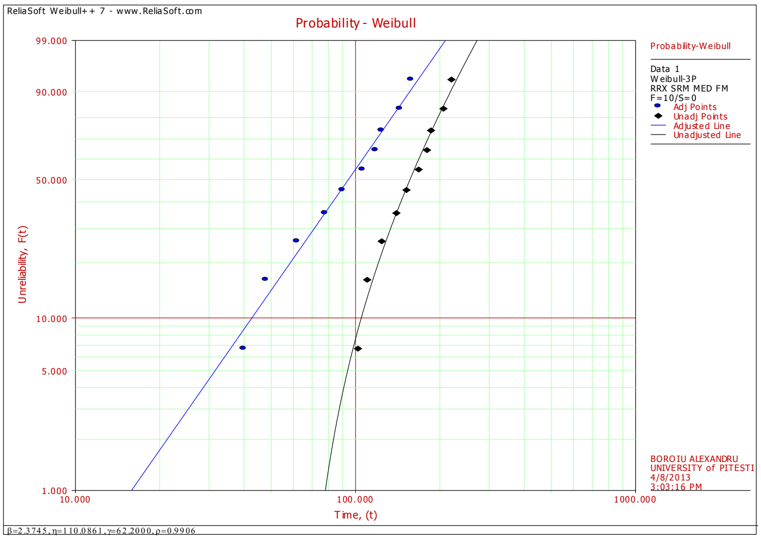

Figure 2. The graph Probability Weibull-3P for the complete test (F = 10, S = 0).

Using the same values for the proper functioning times of the 10 products, it was imagined an incomplete test in which F = 10 and S = 10, but with different scenarios for the values assigned to the 10 monitored elements which are not breaks during the experiment:

A. It is considered censored type test (it ends with the failure of the tenth element, $t_S = t_F = t_{10}$), so for all the 10 elements that continue to operate, there are assigned the value of the last recorded time (F = 10, S = 10, $t_S = t_F = 220$ hours). It is found in this case that the best values for the likelihood rank are for G-Gamma model and Exponential-2P model (figure 3). For the Exponential-2P model it is presented the graph Probability, with a correlation rank $\rho = 0{,}9906$ (figure 4).

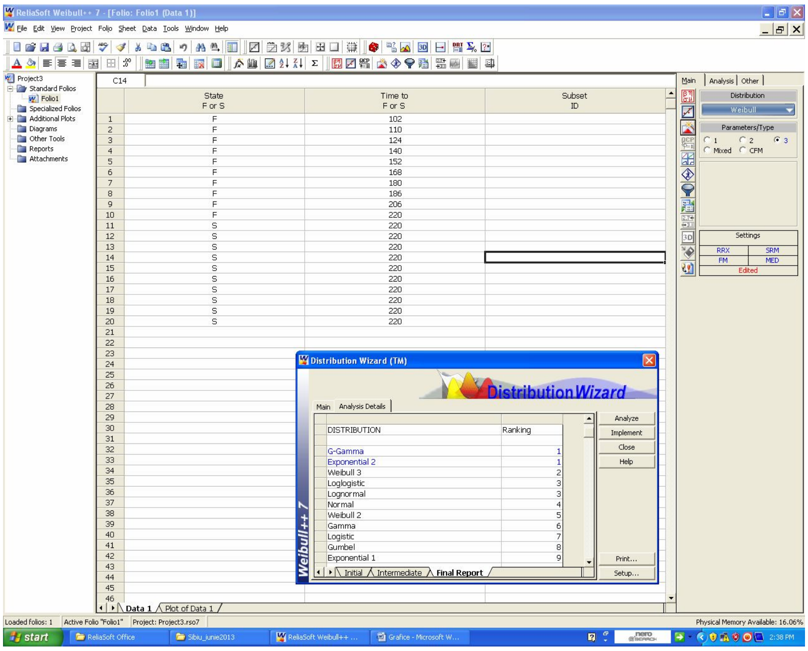

Figure 3. Comparative analysis of censored incomplete patterns obtained for testing (F = 10, S = 10).

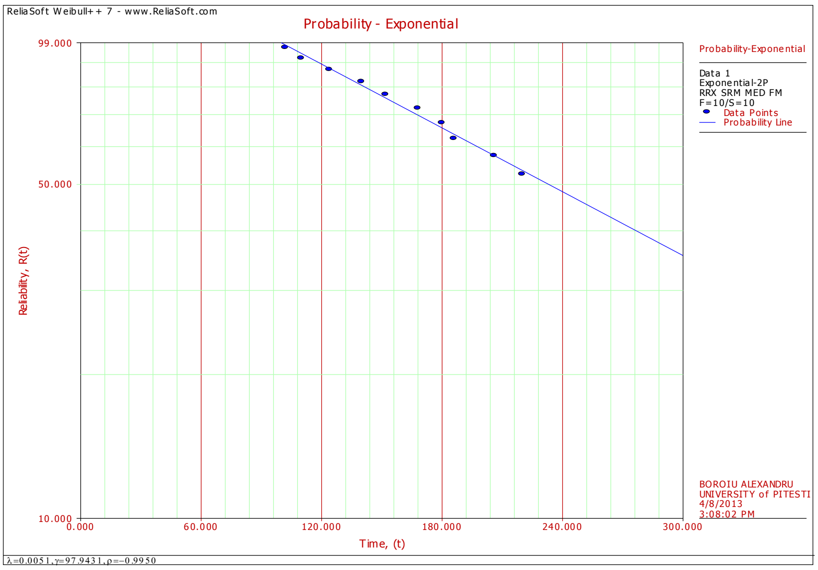

Figure 4. The graph Probability Exponential-2P for the censored test (F = 10, S = 10).

B. It is considered the truncated type test (it not ends with the failure of the tenth element, but at a predetermined time t_S, which is higher than the last recorded time), so for all the 10 elements that continue to operate, there are assigned a value of the time at which the test stops (F = 10, S = 10, $t_S = 300$ hours $> t_F$). It seems that, in this scenario, the modeling program performs the same calculation as for censored test (figure 5): the timeout of the test is considered equal to the time when the last item is damaged (not with the time from the datasheet).

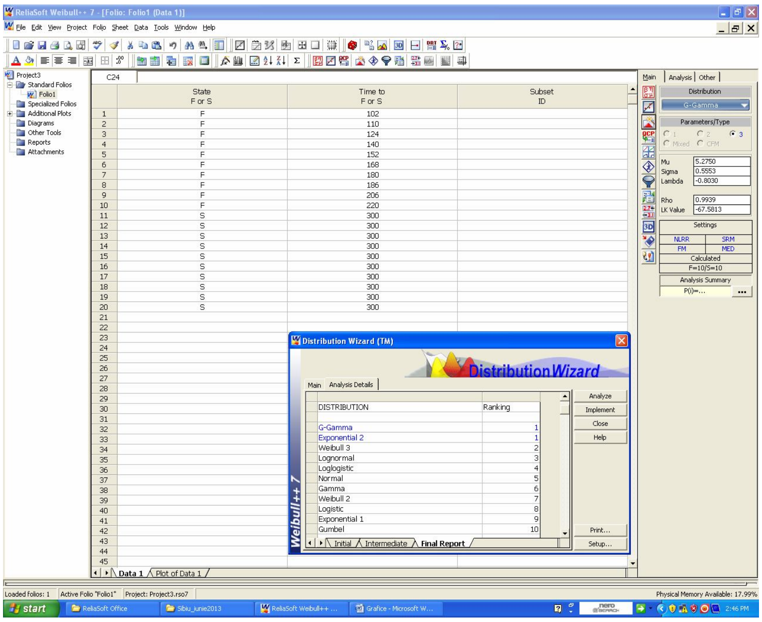

Figure 5. Comparative analysis of truncated incomplete patterns obtained for testing (F = 10, S = 10, t_S = 300 hours).

There are imagined, also, other values for the truncation times of the experiment t_S and it is found that for all these different scenarios there are obtained identical results as incomplete censored tests, i.e. the computing program considers all these different tests as a censored type test (with the censoring time equal to the time at which breaks the tenth element).

C. Continuing the investigations, it is imagined another censored test, in which F = 10, but S = 20 (total, 30 elements are tracked). It appears that this time it is really obtained different models (figure 6), so the program has discriminatory power for censorship tests.

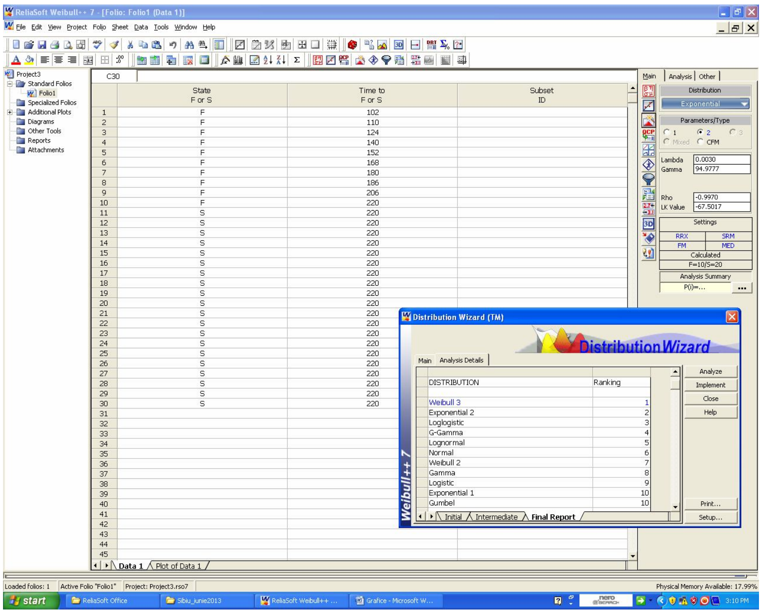

Figure 6. Comparative analysis of truncated incomplete patterns obtained for testing (F = 10, S = 20).

A simple synthetic presentation of relevant issues can be achieved, for example, calculating Exponential 2-P model parameters (with two parameters: failure rate λ and position parameter γ**)** for the presented tests (table 2).

Table 2. Experimental data and results obtained in the framework of reliability tests.

No.	*Test type*	*F*	*S*	*F+S*	*The values of the Exponential-2P model parameters*
1	Complete	10	0	10	λ = 0,0183 hours^{-1}; γ = 102 hours.
2	Censored ($t_S = t_F$ = 220 hours)	10	10	20	λ = 0,0051 hours^{-1}; γ = 97,9431 hours.
3	Truncated (t_S = 300 hours)	10	10	20	**identical with row 2 !**
4	Censored	10	20	30	λ = 0,0030 hours^{-1}; γ = 94,9777 hours.

It is concluded that the computer program correctly identifies the complete test and the incomplete tests of censored type, but not the incomplete tests of truncated type.
As a result of this finding, we intend to perform a research to provide those theoretical elements necessary to identify an incomplete test of truncated type and for creating a suitable computing program to model reliability based on this type of tests.

REALIZED RESEARCHES

To find the theoretical elements necessary for processing the data obtained through incomplete tests of truncated type, it can be started from the most visible reliability indicator of reliability which depends of the type of reliability test, the estimated value of mean time between failures m [1].

• for complete tests:

$$m = \frac{\sum_{1}^{F} t_i}{F} \tag{1}$$

• for incomplete tests of censored type [6, 8]:

$$m_{cenz} \geq \frac{\sum_{1}^{F} t_i + S \cdot t_F}{F + S} \tag{2}$$

• for incomplete tests of truncated type:

$$m_{tr} \geq \frac{\sum_{1}^{F} t_i + S \cdot t_{tr}}{F + S} \tag{3}$$

where:

- t_F is the time corresponding to the failure of the last element in the censored test;
- t_{tr} is the truncation time of the test.

It can be seen that the estimated average for the truncated test m_{tr} can be expressed according to the estimated average for the censored test, m_{cenz}:

$$m_{tr} = m_{cenz} + \frac{S(t_F - t_{tr})}{F + S} \tag{4}$$

This relationship will be useful to correct the mean when found as a parameter in the mathematical model for reliability, between the following:

• the uniform model:

$$R(t) = 1 - \frac{t}{m} \tag{5}$$

• the Exponential-1P model:

$$R(t) = e^{-\lambda \cdot t} = e^{-\frac{t}{m}} \tag{6}$$

• the Exponential-2P model [12]:

$$R(t) = e^{-\lambda \cdot t} = e^{-\frac{t-\gamma}{m-\gamma}} \tag{7}$$

• the normal model (which has two parameters: average m and standard deviation σ):

$$R(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-m)^2}{2\sigma^2}} \tag{8}$$

For the normal model it can correct even the second parameter, the standard deviation σ, through a simple relationship obtained under the "rule of 3σ" [5]:

$$\sigma_{tr} = \sigma_{cenz} + \frac{m_{tr} - m_{cenz}}{3} \tag{9}$$

• the Weibull model: this case requires a more complex analysis. So, in the most general case, for Weibull-3P model, the mean time between failures value m is depending of the all 3 Weibull parameters [2,3]:

$$m = \gamma + \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right) \tag{10}$$

where Γ represents the Euler function of first rank (Gamma type), defined through the analytical relation:

$$\Gamma(x) = \int_0^{+\infty} t^{x-1} \cdot e^{-t} \cdot dt \tag{11}$$

Since the analytic relation of this function is quite complicated, in reliability studies is more easily to work with the function values calculated and listed in tables [10].
The indicator m is in relation to all the three Weibull parameters, so we are not dealing with a bi-univocal relationship, deterministic, so that will be performed an analysis to decide which of the three indicators is most appropriate to be corrected depending on the value of m, and thus depending on the type of test.
For this, we must define the three Weibull parameters [4, 13]:
- γ is the localization parameter or position parameter, an constant that defines the start time of the variation of reliability function R(t);
- η is *the scale parameter,* expresses the extension distribution on the time axis; so, if (t – γ) is equal with η, R(t) becomes:

$$R(t) = e^{-1^{\beta}} = e^{-1} = 0{,}368, \qquad (12)$$

ie, scale parameter represents the time, measured from the moment $\gamma = 0$, at which 63,2% of the elements can be failed. Therefore, this parameter expresses a characteristic operating time.
- β is the shape parameter, it is dimensionless and represents the parameter that determines the shape and curves of variation for the reliability indicators.
The parameters γ and η are expressed in time units and can be graphically highlighted [7, 9] - figure 7.

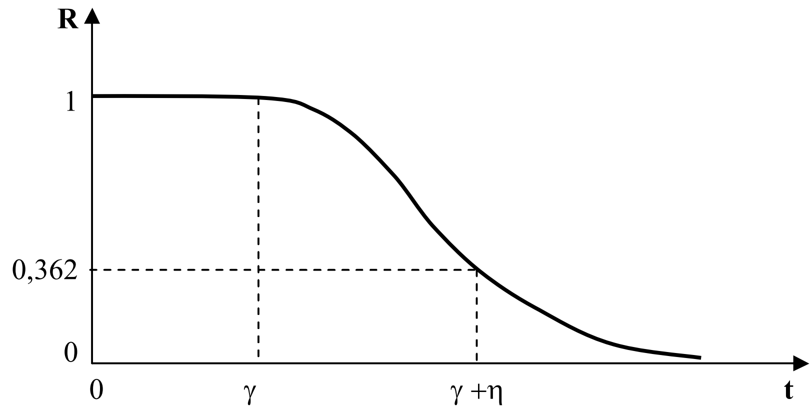

Figure 7. The highlighting of the parameters γ and η on the graph R(t) in the case of Weibull law.

As the previously revealed problem express that the program does not offer the possibility of extending the distribution on the time scale according to the value of truncation time (larger than the censoring time), it follows that the most suitable to be put in a deterministic relationship with the mean time between failures m is precisely the scale parameter η. For this, there will be processed the analytical relations for the mean in the case of the two types of tests – the censored test (for which the program calculates the parameters Weibull, including η_{cens}) and the truncated test (which is intended to determine the parameter η_{tr}):

$$m_{cenz} = \gamma + \eta_{cenz} \cdot \Gamma\left(\frac{1}{\beta} + 1\right) \qquad (13)$$

$$m_{tr} = \gamma + \eta_{tr} \cdot \Gamma\left(\frac{1}{\beta} + 1\right) \qquad (14)$$

It results the inegality:

$$\frac{m_{cenz} - \gamma}{\eta_{cenz}} = \frac{m_{tr} - \gamma}{\eta_{tr}}, \qquad (15)$$

By reducing the inequalities (2) and (3) to equalities there is obtained concrete and satisfactory values for the means, so based on the relation (15) it can be effectively realized the calculation for parameter η_{tr}:

$$\eta_{tr} = \eta_{cenz} \cdot \frac{m_{tr} - \gamma}{m_{cenz} - \gamma} = \eta_{cenz} \cdot \frac{\dfrac{\sum_{1}^{F} t_i + S \cdot t_{tr}}{F + S} - \gamma}{\dfrac{\sum_{1}^{F} t_i + S \cdot t_F}{F + S} - \gamma} \qquad (16)$$

Thus, in the case of truncated test from the position 3 in Table 2, are obtaining using the computer program 3-P Weibull model values of three parameters: β = 1,0164; η = 205,7008 hours; γ = 94,0750 hours. Applying the relation (16), on obtain the corrected value for the parameter η:

$$\eta_{tr} = \eta_{cenz} \cdot \frac{\dfrac{\sum_{1}^{F} t_i + S \cdot t_{tr}}{F+S} - \gamma}{\dfrac{\sum_{1}^{F} t_i + S \cdot t_F}{F+S} - \gamma} = 205{,}7008 \cdot \frac{\dfrac{\sum_{1}^{10} t_i + 10 \cdot 300}{20} - 94{,}0750}{\dfrac{\sum_{1}^{10} t_i + 10 \cdot 220}{20} - 94{,}0750} = 293{,}3963 hours,$$

a value according to what is expected for the truncated test: a more extended theoretical distribution on the time axis, compared with the case of censored test [11].

Therefore, the Weibull model which will be used for the truncated test from the position 3 in Table 2 will have the parameters: β = 1,0164; η = 293,3963 hours (higher that initial value: η = 205,7008 ore); 94,0750 hours.

CONCLUSIONS

For the data from warranty period, for each of the models considered can build a computing program that is used for the case of incomplete tests of truncated type, so:

- relation (4): for the models who have a mean like parameter (uniform model, Exponenţial-1P model, Exponenţial-2P model and normal model);
- relation (4) and (9): for the normal model;
- relation (16): for the Weibull-2P model and Weibull-3P model

For other mathematical models (including the single-parametric, even if not appropriate parameter choise that will correct), analysis can be performed to create suitable calculation program for modeling reliability when testing truncated

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