THE GENERAL FORM OF THE EQUATION OF MOTION FOR PLANAR MECHANICAL SYSTEMS THAT ARE COMPOSED WITH JOINTS WITH AND WITHOUT CLEARANCE

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ABSTRACT - Kinematic couplings clearances modified by changing the kinematic chain motion laws of motion of elements and shocks that may occur. The paper presents the general form of the differential equation of motion obtained using multibody method for planar kinematic chains with rotational kinematic couplings with clearances

INTRODUCTION

Establish the work of a new form of the matrix constraints made possible the development multibody method that allows the numerical study of the general movement of articulated planar systems games and numerical study of the positions of equilibrium and vibrations of these systems

DYNAMIC EQUATIONS OF MOTION

<u>If no clearances rotation kinematic couplings</u>

By isolating the element with the sequence number j, taking into account the principle of action and reaction, noting the points O_k, O_h obtained with the reaction H_k, V_k, H_h, V_h represented in figure 1.

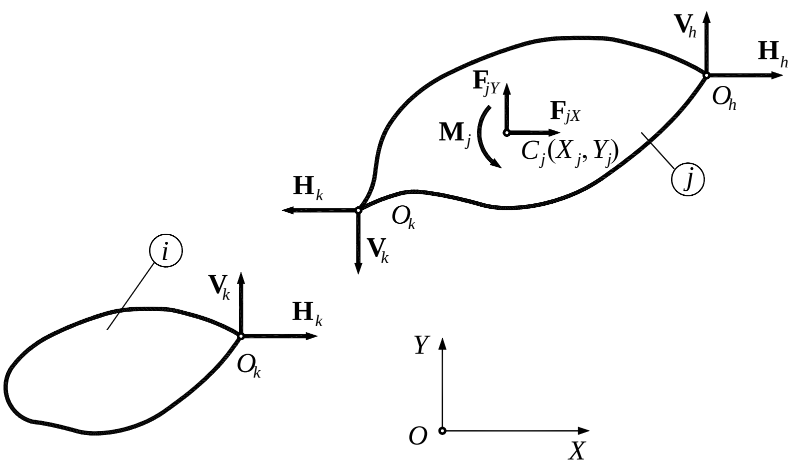

Figure 1. Isolation item with sequence number of cinematic couplings j rotation without clearance

Considering the notation:

- m_j, mass element;
- J_j, moment of inertia to the center of gravity C_j;
- F_{jX}, F_{jY}, projections on the axes OX, OY of the resultant external force acting on the item;
- M_j, time resulting from point C_j, the momentum theorem [1] to obtain equations

$$m_j \ddot{X}_j = F_{jX} - H_k + H_h, m_j \ddot{Y}_j = F_{jY} - V_k + V_h, \tag{1}$$

and the angular momentum theorem [1] to point C_j to obtain the

$$J_j \ddot{\theta}_j = M_j + H_k \cdot U_{kY}^{(j)} - V_k \cdot U_{kX}^{(j)} - H_h \cdot U_{hY}^{(j)} + V_h \cdot U_{hX}^{(j)} \tag{2}$$

Equations (1), (2) by using the notations

$$\left[m_j\right] = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & J_j \end{bmatrix}, \tag{3}$$

$$\left\{F_j\right\} = \left[F_{jX} \quad F_{jY} \quad M_j\right]^T, \tag{4}$$

$$\left\{R_k\right\} = \left[H_k \quad V_k\right]^T, \left\{R_h\right\} = \left[H_h \quad V_h\right]^T \tag{5}$$

and taking into account the first relation (3.5), is written as matrix

$$\left[m_j\right]\left\{\ddot{q}_j\right\} = \left\{F_j\right\} - \left[B_k^{(j)}\right]^T \left\{R_k\right\} + \left[B_h^{(j)}\right]^T \left\{R_h\right\} \tag{6}$$

Based on relation (6) that for a plan to obtain, with notations,

$$[m] = \begin{bmatrix} [m_1] & [0] & \dots \\ [0] & [m_2] & \dots \\ \dots & \dots & \dots \end{bmatrix}, \tag{7}$$

$$\{q\} = \left[\{q_1\}^T \quad \{q_2\}^T \quad \dots\right]^T, \tag{8}$$

$$\{F\} = \left[\{F_1\}^T \quad \{F_2\}^T \quad \dots\right]^T, \tag{9}$$

$$\{R\} = \left[\{R_1\}^T \quad \{R_2\}^T \quad \dots\right]^T, \tag{10}$$

equation

$$[m]\{\ddot{q}\} = \{F\} - [B]^T \{R\}, \tag{11}$$

where $[B]$ is the matrix of constraints.

<u>If the clearance rotation kinematic couplings</u>

By isolating the element with the serial number j, taking into account the principle of action and reaction, noting the reactions of N_k, N_h points $O_k^{(j)} O_h^{(j)}$, we get the representation in figure 2.

From theorem momentum equations are obtained;

$$\begin{aligned} m_j \ddot{X}_j &= F_{jX} - N_k D_{kX} + N_h D_{hX}, \\ m_j \ddot{Y}_j &= F_{jY} - N_k D_{kY} + N_h D_{hY}, \end{aligned} \tag{12}$$

theorem and the angular momentum equation is obtained

$$J_j \ddot{\theta}_j = M_j - N_k \left(-D_{kX} U_{kY}^{(j)} + D_{kY} U_{kX}^{(j)}\right) + N_h \left(-D_{hX} U_{hY}^{(j)} + D_{hY} U_{hX}^{(j)}\right) \tag{13}$$

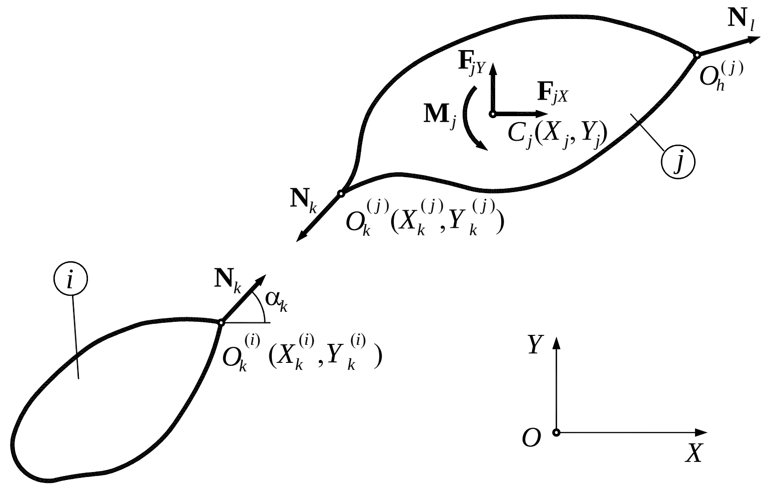

Figure 2. Isolation item with serial number j, with clearance joint.

Equations (12), (13) taking into account the previous notation, meet in the equation matrix

$$\left[m_j\right]\left\{\ddot{q}_j\right\}=\left\{F_j\right\}-N_k\left[E_k^{(j)}\right]^T+N_h\left[E_h^{(j)}\right]^T \tag{14}$$

Matrix equation (14) is formally identical to equation (6) the matrices $\left[B_k^{(j)}\right]$ are replaced with matrices $\left[E_k^{(j)}\right]$

<u>Differential equations of motion matrix equation system</u>

If point O_k is a kinematic coupling of rotation with the clearance, and the point O_h is a kinematic coupling of rotation without clearance, then, obviously, we obtain the equation

$$\left[m_j\right]\left\{\ddot{q}_j\right\}=\left\{F_j\right\}-N_k\left[E_k^{(j)}\right]^T+\left[B_h^{(j)}\right]\left\{R_h\right\}, \tag{15}$$

if, for example, point O_k is required motion, and the point O_h is a kinematic coupling of rotation with the clearance, then get the matrix equation

$$\left[m_j\right]\left\{\ddot{q}_j\right\}=\left\{F_j\right\}-\left[\widetilde{B}_k^{(j)}\right]\left\{R_k\right\}+N_h\left[E_h^{(j)}\right]^T, \tag{16}$$

Thus the system of figure 3, which is rotating element, the elements are driven only by their own weights and O_3 rotational kinematic coupling is the clearance matrix differential equation is

$$[m]\{\ddot{q}\}=\{F\}-[B]^T\{R\}, \tag{17}$$

where

$$[m]=\begin{bmatrix} [m_2] & [0] & [0] \\ [0] & [m_3] & [0] \\ [0] & [0] & [m_4] \end{bmatrix}, \tag{18}$$

$$\{q\}=\left[\{q_2\}^T \quad \{q_3\}^T \quad \{q_4\}^T\right]^T, \tag{19}$$

$$\{F\}=\left[0 \quad -m_2 g \quad 0 \quad 0 \quad -m_3 g \quad 0 \quad 0 \quad -m_4 g \quad 0\right]^T, \tag{20}$$

$$\{R\}=\left[H_2 \quad V_2 \quad N_3 \quad H_4 \quad V_4 \quad H_5 \quad V_5\right]^T, \tag{21}$$

$$[B]=\begin{bmatrix} [B_2^{(2)}] & [0] & [0] \\ -[E_3^{(2)}] & [E_3^{(3)}] & [0] \\ [0] & -[B_4^{(3)}] & [B_4^{(4)}] \\ [0] & [0] & -[B_5^{(4)}] \end{bmatrix}, \tag{22}$$

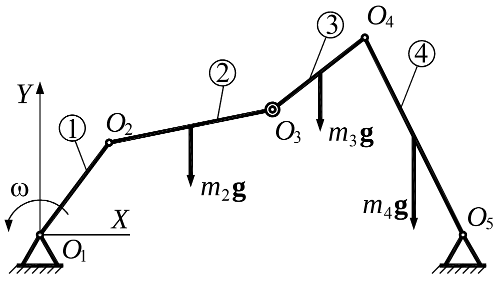

Figure 3. Planar system of bars at which the element 1 has uniform rotational motion, the joint O_3 is with clearance, and the elements are acted only by their own weights.

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At the matrix equation (17) the following equality

$$[m]\{\dot{q}\}=\{C\}, \tag{23}$$

where

$$\{C\}=l_1\omega[-\sin\omega t \quad \cos\omega t \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T , \tag{24}$$

By derivation with respect to time of the relationship (23) and equation (17) to obtain the general matrix equation of motion of multibody systems

$$\begin{bmatrix} [m] & [B]^T \\ [B] & [0] \end{bmatrix}\begin{bmatrix} \{\ddot{q}\} \\ \{R\} \end{bmatrix}=\begin{bmatrix} \{F\} \\ \{\dot{C}\}-[\dot{B}]\{\dot{q}\} \end{bmatrix}, \tag{25}$$

where matrix $\{R\}$ of the reactions is actually a matrix with sign changed Lagrange's multipliers

APPLICATION

For the mechanism designed in fig. 4 we examine its movement, knowing that the O_2, O_3 points are clearance pin joints, the element 1 has a rotational motion with constant angular speed ω, and the initial position being represented in the figure.
Also, the following numerical data are known: $l_1 = 0.1\,m$, $l_2 = 0.28\,m$, $l_3 = 0.3\,m$, $m_2 = m_3 = 2\,kg$, $J_2 = J_3 = 0.06\,kgm^2$, $\omega = 10\,rad/s$, $M = -20\,Nm$, $r_2 = r_3 = 0.02\,m$.

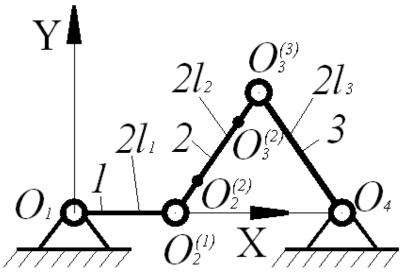

Figure 4. Application.

The initial conditions are: $t = 0\,s$, $X_2 = 0.35\,m$, $Y_2 = 0.15\sqrt{3}\,m$, $\theta_2 = 60^\circ$, $X_3 = 0.65\,m$, $Y_3 = 0.15\sqrt{3}\,m$, $\theta_3 = -60^\circ$, $\dot{X}_2 = 0\,m/s$, $\dot{Y}_2 = 0.2\omega\,m/s$, $\dot{\theta}_2 = 0\,rad/s$, $\dot{X}_3 = 0.05\omega\sqrt{3}\,m/s$, $\dot{Y}_3 = 0.05\omega\,m/s$, $\dot{\theta}_3 = -\dfrac{\omega}{3}\,rad/s$.

Using the notations

$$D_1^{(2)} = \frac{X_2 - l_2\cos\theta_2 - 2l_1\cos\omega t}{r_2},\ D_2^{(2)} = \frac{Y_2 - l_2\sin\theta_2 - 2l_1\sin\omega t}{r_2}, \tag{26}$$

$$D_3^{(2)} = \left(D_1^{(2)}\sin\theta_2 - D_2^{(2)}\cos\theta_2\right)l_2,\ C_1^{(2)} = -2l_1\omega\left(D_1^{(2)}\sin\omega t - D_2^{(2)}\cos\omega t\right) \tag{27}$$

$$D_1^{(3)} = \frac{X_3 - l_3\cos\theta_3 - X_2 - l_2\cos\theta_2}{r_3},\ D_2^{(3)} = \frac{Y_3 - l_3\sin\theta_3 - Y_2 - l_2\cos\theta_2}{r_3}, \tag{28}$$

$$D_3^{(3)} = \left(D_1^{(3)}\sin\theta_2 - D_2^{(3)}\cos\theta_2\right)l_2,\ D_4^{(3)} = \left(D_1^{(3)}\sin\theta_3 - D_2^{(3)}\cos\theta_3\right)l_3, \tag{29}$$

$$\{\mathbf{q}\} = \begin{pmatrix} X_2 & Y_2 & \theta_2 & X_3 & Y_3 & \theta_3 \end{pmatrix}^T,\ \{\mathbf{F}\} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & M_3 \end{pmatrix},$$
$$\{\mathbf{C}\} = \begin{pmatrix} C_1^{(2)} & 0 & 0 & 0 \end{pmatrix}^T, \tag{30}$$

$$\dot{D}_1^{(2)} = \frac{\dot{X}_2 - l_2\dot{\theta}_2\sin\theta_2 + 2l_1\omega\sin\omega t}{r_2}, \tag{31}$$

$$\dot{D}_2^{(2)} = \frac{\dot{Y}_2 - l_2\dot{\theta}_2\cos\theta_2 - 2l_1\omega\cos\omega t}{r_2}, \tag{32}$$

$$\dot{D}_3^{(2)} = \left[\dot{D}_1^{(2)}\sin\theta_2 - \dot{D}_2^{(2)}\cos\theta_2 + \left(D_1^{(2)}\cos\theta_2 - D_2^{(2)}\sin\theta_2\right)\dot{\theta}_2\right]l_2, \tag{33}$$

$$\dot{C}_1^{(2)} = -2l_1\omega\left[\dot{D}_1^{(2)}\sin\omega t - \dot{D}_2^{(2)}\cos\omega t + \left(D_1^{(2)}\cos\omega t - D_2^{(2)}\sin\omega t\right)\omega\right], \tag{34}$$

$$\dot{D}_1^{(3)} = \frac{\dot{X}_3 + l_3\dot{\theta}_3\sin\theta_3 - \dot{X}_2 + l_2\dot{\theta}_2\sin\theta_2}{r_3}, \tag{35}$$

$$\dot{D}_2^{(3)} = \frac{\dot{Y}_3 - l_3\dot{\theta}_3\cos\theta_3 - \dot{Y}_2 - l_2\dot{\theta}_2\cos\theta_2}{r_3}, \tag{36}$$

$$\dot{D}_3^{(3)} = \left[\dot{D}_1^{(3)}\sin\theta_2 - D_2^{(3)}\cos\theta_2 + \dot{\theta}_2\left(D_1^{(3)}\cos\theta_2 + D_2^{(3)}\sin\theta_2\right)\right]l_2, \tag{37}$$

$$\dot{D}_4^{(3)} = \left[\dot{D}_1^{(3)}\sin\theta_3 - \dot{D}_2^{(3)}\cos\theta_3 + \dot{\theta}_3\left(D_1^{(3)}\cos\theta_3 + D_2^{(3)}\sin\theta_3\right)\right]l_3, \tag{38}$$

$$[\mathbf{B}] = \begin{bmatrix} D_1^{(2)} & D_2^{(2)} & D_3^{(2)} & 0 & 0 & 0 \\ -D_1^{(3)} & -D_2^{(3)} & D_3^{(3)} & D_1^{(3)} & D_2^{(3)} & D_4^{(3)} \\ 0 & 0 & 0 & -1 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 0 & 0 & -1 & -l_3\sin\theta_3 \end{bmatrix}, \tag{39}$$

$$
[\dot{\mathbf{B}}]=\begin{bmatrix} \dot{D}_1^{(2)} & \dot{D}_2^{(2)} & \dot{D}_3^{(2)} & 0 & 0 & 0 \\ -\dot{D}_1^{(3)} & -\dot{D}_2^{(3)} & \dot{D}_3^{(3)} & \dot{D}_1^{(3)} & \dot{D}_2^{(3)} & \dot{D}_4^{(3)} \\ 0 & 0 & 0 & 0 & 0 & l_3\dot{\theta}_3\cos\theta_3 \\ 0 & 0 & 0 & 0 & 0 & -l_3\dot{\theta}_3\sin\theta_3 \end{bmatrix}, \tag{40}
$$

$$
\{\dot{\mathbf{C}}\}=\left(\dot{C}_1^{(2)} \quad 0 \quad 0 \quad 0\right)^T, \ \{\mathbf{R}\}=(N_2 \quad N_3 \quad H_4 \quad V_4)^T, \tag{41}
$$

$$
\{\mathbf{Q}\}=\{\dot{\mathbf{C}}\}-[\dot{\mathbf{B}}]\{\dot{\mathbf{q}}\}, \tag{42}
$$

$$
[\mathbf{M}]=\begin{bmatrix} m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_3 \end{bmatrix}, \tag{43}
$$

the next system is obtained

$$
\begin{bmatrix} [m] & [B]^T \\ [B] & [0] \end{bmatrix}\begin{bmatrix} \{\ddot{q}\} \\ \{R\} \end{bmatrix}=\begin{bmatrix} \{F\} \\ \{\dot{C}\}-[\dot{B}]\{\dot{q}\} \end{bmatrix}, \tag{44}
$$

For our numerical example the results are captured in the diagrams of fig. 5, 6, 7, 8, 9, 10.

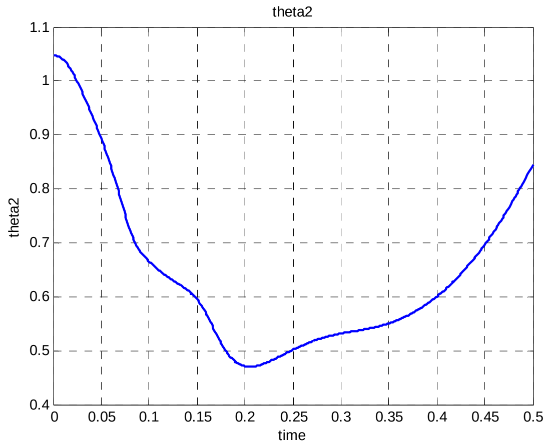

Figure 5. The variation $\theta_2=\theta_2(t)$ for $0\le t\le 0.5\,s$.

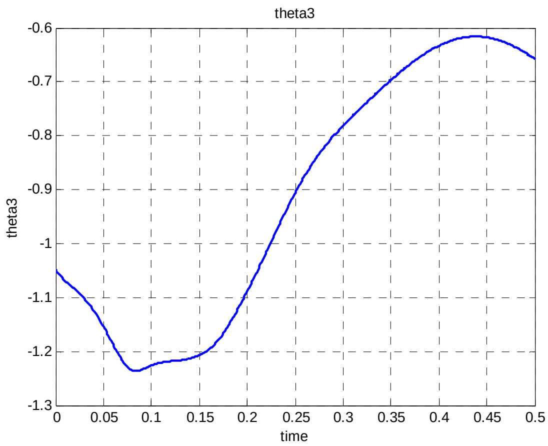

Figure 6. The variation $\theta_3=\theta_3(t)$ for $0\le t\le 0.5\,s$.

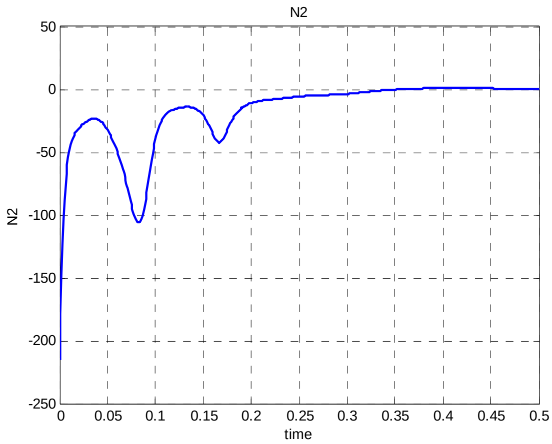

Figure 7. The variation $N_2 = N_2(t)$ for $0 \le t \le 0.5\,s$.

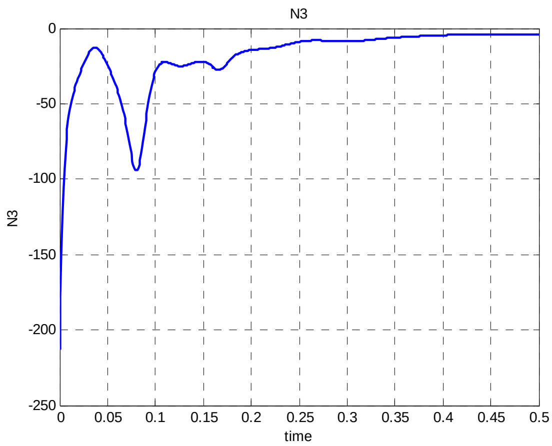

Figure 8. The variation $N_3 = N_3(t)$ for $0 \le t \le 0.5\,s$.

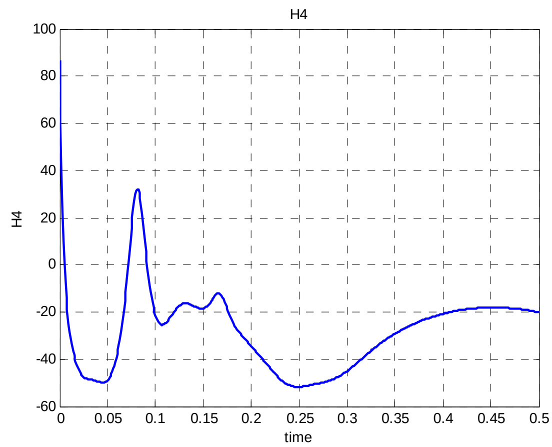

Figure 9. The variation $H_4 = H_4(t)$ for $0 \le t \le 0.5\,s$.

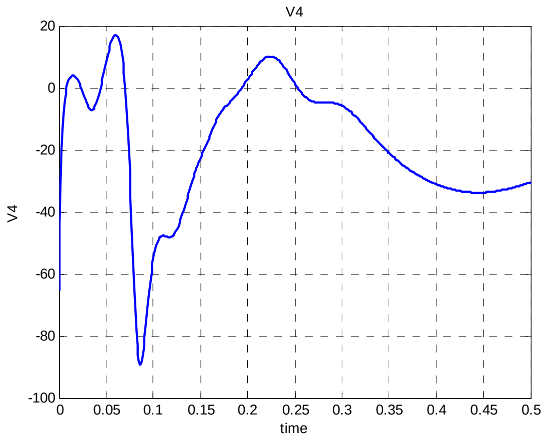

Figure 10. The variation $V_4 = V_4(t)$ for $0 \le t \le 0.5\,s$.

From the diagram presented the reader can easily observe the non-periodical character of the variations for different parameters. Each diagram starts with a transitory period for about $0.2\,s$ when the motion has a random aspect. After this period, the diagrams start to look smooth. The start period can be considered to be characteristic to a many degrees of mobility mechanism because of the clearances.

CONCLUSIONS

In our paper we presented a general multibody type method to study the planar mechanisms with clearances. We obtained the constraints matrix in the most general case and the matrix differential equation of motion. Finally a complete application was solved. The clearances in the joints lead to a quasi-chaotic behavior of motion. The study of this motion will be the goal of our future work.

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