PRODUCTION AND MAINTENANCE POLICIES OPTIMIZATION UNDER A WITHDRAWAL RIGHT CONSTRAINT

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ABSTRACT

In this paper, we deal with the problem of joint maintenance and production under a withdrawal right constraints. The manufacturing system consists of a machine M that produces a single product in order to satisfy a random demand d under given service level constraint.

We first establish an optimal production plan which minimizes the total inventory and production cost taking into consideration the withdrawal right constraints. Secondly, using this optimal production plan we derive an optimal maintenance plan which minimizes the total maintenance cost. Finally, a numerical example is studied in order to apply the developed approach.

KEYWORDS

Random demand, service level, stochastic model, withdrawal right, maintenance strategy, production policy

INTRODUCTION

An integrated approach of maintenance and production policies has recently become an important research area. In deed the development of an optimal production/maintenance plan which minimizes the total cost including production, inventory and maintenance is among the important actions of a hierarchical decision manufacture process. Surely, this is complex to solve since the various uncertainties; the knowing of the random demand evolution by period, the material availability and Failures rate variation. On the other hand, maintenance becomes even more significant with the production control policies implementation like Just-in-Time, which require the availability of machines at the right time. In this context, Abdelnour et al. [1], and Chan et al. [3] proposed a simulation model to evaluate the performance of a production line operating in push system. Rezg and Al. [7] presented a common optimization of the preventive maintenance and stock control in a production line made up of *N* machines. In the same context of integrating maintenance and production Rezg and al.[8] presented a mathematical model and a numerical procedure which allows determining a joint optimal inventory control and age based preventive maintenance policy for a randomly failing production system. New maintenance/production strategies by taking into account the context of subcontractor are studied by Dellagi, S. et al. [4]. They developed and optimize a new maintenance policy with taking into account a machine subcontractor constraint and they studied the case of several subcontractor machines and developed an optimal switching strategy between the subcontractor machines.

In the traditional approach, which dissociates maintenance and production, the demand is assumed known, constant and within an infinite horizon. Whereas in our study, the demand is random on a finite time horizon. To meet such a demand while minimizing production and inventory costs, it is necessary to vary the production rate. In reality, the failure rate increases with time and according to the use of the equipment. It is obvious, when we produce more, we degrade more the machine. Moreover, a change in production rate can also be beneficial to reach production goals when unpredicted events happen in the system that disturbs the original production plan. In the literature, the consideration of the equipment failure according to the production rate is rarely studied. Among these works, we can cite Hu et al. [6] who discussed the conditions of optimality of the hedging point policy for production systems in which the failure rate of machines depends on the production rate. Recently Hajej et al. [5] dealt with combined production and maintenance plans for a manufacturing system satisfying a random demand over a finite horizon. In their model, they assumed that the failure rate depends on the time and the production rate.

In the same context of Hajej et al, we will study a problem of a jointly production/maintenance policy for a manufacturing system take into account the return of products by the customer. In fact our approach consists at a manufacturing system composed of one machine which produces a single product in order to satisfy a random demand. In reality, the objective is to establish sequentially the economical production plan and the optimal maintenance strategy. That's why, in order to establish an optimal maintenance strategy, we need to take into account the influence of withdrawal right on the production plan and the influence of the economical production plan (this last) on the material failure rate and the average number of failure.

The paper is organized as follows. Section 2 describes the problem and the working assumptions. Section 3 presents the problem formulation. In section 4, considering the influence of the production plan in the manufacturing system degradation, we developed an analytical model for evaluating maintenance and production. In section 5, we presented a numerical example, in order to apply the analytical results. Finally we conclude in section 6.

MANUFACTURING PROBLEM

We are concerned with the problem of the optimal production planning problem formulation of a manufacturing system composed of machine *M* which produces a single product in order to meet the random demand *d* law characterized by a Normal distribution facing the system at a minimum cost. The Normal mean and the standard deviation parameters are respectively $\hat{d}$ and σ_d^2.

This study treated the manufacturing case where the production system necessity to satisfy the random demand with inventory service level θ. The step, to obtain an economical production plan, is taking into account the products returned by the customer who are still new and in stock. In the manufacturing field, the back of the products to stock, called the withdrawal right. This right gives the customer a specific deadline for returning products.

The problem is illustrated in Figure.1.

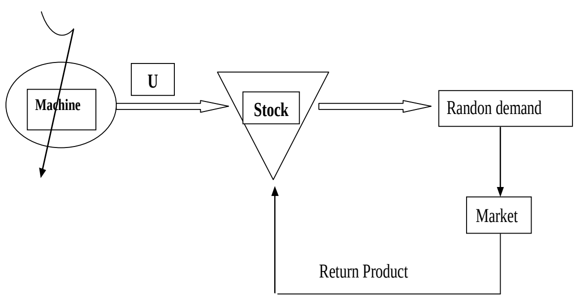

Fig.1. Problem description

The machine *M* is subject to a random failure. The probability degradation law of machine *M* is described by the probability density function of time to failure *f(t)*and for which the failure rate $\lambda(t)$ increases with time and according to the production rate $u(t)$. Failures of machine *M* can be reduced through preventive maintenance activities. Preventive maintenance (PM), usually scheduled periodically at certain time intervals, is a policy aimed at improving the overall reliability of a system.

Our first objective is to establish an economical production plan take into account the withdrawal right and satisfying the randomly demand. Secondly, using the optimal production plan obtained, we determined the optimal preventive maintenance period. The use of the optimal production plan in maintenance study is justified by the influence of the production rate on the deterioration of the manufacturing system.

PROBLEM FORMULATION

NOTATION

C_{pr} : unit production cost
C_s : holding cost of a product unit during the period *k*
f(t) : probability density function of time to failure for the machine
H : the finite production horizon
M_p : preventive maintenance action cost
M_c: corrective maintenance action cost
mu : monitory unit
$R_i(t)$: Reliability function
S(k) : The stock level at the end of the period *k*
U_{max} : the maximal production rate
u(k) : The production rate for period *k*
r(k) : The product returned by the customer for period *k*
G : the total expected cost including production and inventory over a finite horizon *H*.
ζ_p : the maintenance total cost expected per time unit
θ : Probabilistic index; related to the customer satisfaction
δ: The return of product rate
τ: The return of product Delay
Δt : period length

LINEAR STOCHASTIC PROBLEM

The idea is to minimize the expected production and holding costs over a finite time horizon *[0, H]*. It's assumed that the horizon is portioned equally into *H* periods. The demand is satisfied at the end of each period. Let $\{g_k, k=1,....., H\}$ represent holding and production costs (they will be formulated in the next subsection), and *E{}* denotes the mathematical expectation operator.

The following aggregate sequential stochastic programming problem provides an optimal production plan over the planning horizon:

$$\underset{u_k}{Min} \; E\left\{ g_H(S_H) + \sum_{k=0}^{H-1} g_k\left(S_k, u_k\right)\right\}$$

Subject to:

$$S(k+1)=S(k)+r(k)+u(k)-d(k) \quad k=0,1,.....,H-1 \tag{1}$$

$$r(k)=\delta\times d(k-\tau) \tag{2}$$

$$\text{Prob}\;[S(k+1)\geq 0]\geq\theta \qquad k=0,1,.....,\; H-1 \tag{3}$$

$$0 \le u_k \le u_{\max} \quad k = 0,1,\ldots, H-1 \tag{4}$$

Constraint (1) defines the inventory balance equation for each time period. The relation (2) defines the product quantity returned by the customer; this quantity is part of the demand returned by the customer after the specific deadline τ. The constraint (2) imposes the service level requirement for each period as well as a lower bound on inventory variables so as to prevent stockouts. Finally, the last constraint defines an upper bound on the production level during each period *k* cannot exceed a given maximal production rate U_{max}.

The purpose is to develop and optimize the expected production and holding costs over the finite time horizon *H*. As mentioned above, the demand must be satisfied at the end of each period, the problem can be formulated as a stochastic optimal control problem under a threshold inventory level constraint. The system model is described by a hybrid state with continuous component, the dynamic of the stock.

The state equation of the stock level is given by:

$$S(k+1) = S(k) + r(k) + u(k) - d(k) \quad k = 0,1,\ldots, H-1$$

With $s_1(0) = s_0$, where s_0 is the given initial inventory.

The expected cost including production and holding costs for the period *k* is given by:

$$g_k(u_k, s_k) = C_S \cdot E\left\{\left(S(k)^2\right)\right\} + C_{pr} \cdot \left(u(k)^2\right) \tag{5}$$

Remark:

The use of quadratic cost that allows penalizing both excess and shortage of inventory level

Since that the total expected cost including production and inventory over a finite *H* periods is expressed by:

$$G(u) = \sum_{k=0}^{H} g_k(u(k), S(k)) = C_s \times E\left\{\left(S(H)^2\right)\right\} + \sum_{k=0}^{H-1}\left[C_s \times E\left\{S(k)^2\right\} + C_{pr} \times u(k)^2\right] \tag{6}$$

Remark:

$C(U(H))^2$ is not included in the cost formulation because we don't consider the production command at the end of the horizon *H*.

Thus, our problem is defined:

$$\min_u \left[C_S \times E\left\{\left(S(H)^2\right)\right\} + \sum_{k=0}^{H-1}\left[C_S \times E\left\{\left(S(k)^2\right)\right\} + C_{pr} \times \left(u(k)^2\right)\right]\right]$$

with

$$S(k+1) = S(k) + r(k) + u(k) - d(k) \quad k = 0,1,\ldots, H-1$$

$$r(k) = \delta \times d(k-\tau)$$

$$\text{Prob}\left[S(k+1) \ge 0\right] \ge \theta \quad k = 0,1,\ldots, H-1$$

$$0 \le u_k \le u_{\max} \quad k = 0,1,\ldots, H-1$$

MAINTENANCE POLICY

We seek to optimize the cost model associated with the preventive maintenance with minimal repair policy derived above. We recall that we assume that the failure rate $\lambda(t)$ is increasing in both time and production rate u(t). Since the machine production rate is variable over the horizon $H.\Delta t$, the degradation will be variable too.

Consequently, the objective here is to take into account the production rate in determining the partitions optimal number N^* of preventive maintenance actions to be carried out, which in turn means that the preventive maintenance action takes place at $T^* = H/N^*$ tu(time unit).

Thus, the total maintenance cost over *H* is given by:

$$\varphi(N,U) = (N-1)\times M_p + M_c \times \chi(U,N) \quad (7)$$

With
- $\varphi(N,U)$ is the maintenance cost related to the production plan.
- N is the number of PM actions during the time horizon $H\Delta t$.
- $\chi(U,N)$ the average number of failure

ANALYTICAL STUDY

PRODUCTION POLICY

In this section, we determine a deterministic formulation in order to easier the resolution of our stochastic problem.

a) Production and holding costs:

We can simplify the expected value of the production/inventory costs of eq. (6) as follows:

Lemma1:

$$G(u) = C_s \cdot \left(\hat{S}(H)\right)^2 + \sum_{k=0}^{H-1}\left[C_s \cdot \left(\hat{S}(k)\right)^2 + C_{pr} \cdot u(k)^2\right] + C_s \times \sigma_d^2 \times \frac{(H-1)}{2} \times \left[H + \delta^2 \times (H-2)\right] \quad (8)$$

Where $\hat{s}_1(k)$ represents level of mean principle stock at the end of period *k*.

b) The inventory balance equation:

Letting $d_k = \hat{d}_k$, the state balance equation of stock (eq.1) can be converted to:

$$\hat{S}(k+1) = \hat{S}(k) + u(k) + r(k) - \hat{d}(k)$$
$$r(k) = \delta \times \hat{d}(k-\tau) \quad (9)$$

c) The service level constrain:

The transformation of the service level constraint in a deterministic form is obtained by specifying certain minimum cumulative production quantities that depend on the service level requirements. This transformation is given by the next lemma:

Lemma 2:

$$\text{Prob}(s(k+1)\geq 0)\geq \theta \Rightarrow (U(k)\geq U_\theta(s(k),\theta)) \qquad k=0,1,\ldots,H-1 \quad (10)$$

With:

$U_\theta(\ )$: Minimum cumulative production quantity

$$U_\theta(s(k),\theta) = \left(V_{d,k}\times V_{d,k-\tau}\right)\times\varphi^{-1}(\theta) + \hat{d}(k) - \delta\times\hat{d}(k-\tau) - S(k) \qquad k=0,1,\ldots,H-1$$

$V_{d,k}$: Variance of demand *d* at period *k*

$V_{d,k-\tau}$: Variance of demand d at period *k*-τ

$\phi_{d,k}$: Cumulative Gaussian distribution function with mean $\left(\frac{1}{V_{d,k-\tau}}\times\hat{d}(k) - \frac{\delta}{V_d}\times\hat{d}(k-\tau)\right)$ and finite variance $\left(\left(\frac{1}{V_{d,k-\tau}}\right)^2\times V_{d,k} + \left(-\frac{\delta}{V_d}\right)^2\times V_{d,k-\tau} \geq 0\right)$

$\phi_{d,k}^{-1}$: Inverse distribution function

Proof:

$$S(k+1) = S(k) + r(k) + u(k) - d(k)$$

$$\Pr ob\left(S\left(k+1\right)\geq 0\right)\geq \theta$$

$$\Pr ob\left(S\left(k\right)+r\left(k\right)+u\left(k\right)-d\left(k\right)\geq 0\right)\geq \theta$$

$$\Pr ob\left(S\left(k\right)+r\left(k\right)+u\left(k\right)\geq d\left(k\right)\right)\geq \theta$$

$$\Pr ob\left(S\left(k\right)+r\left(k\right)+u\left(k\right)-\hat{d}\left(k\right)\geq d\left(k\right)-\hat{d}\left(k\right)\right)\geq \theta$$

$$\Pr ob\left(d\left(k\right)-\hat{d}\left(k\right)\leq S\left(k\right)+r\left(k\right)+u\left(k\right)-\hat{d}\left(k\right)\right)\geq \theta$$

$$\Pr ob\left(d\left(k\right)-\hat{d}\left(k\right)\leq S\left(k\right)+\delta\times d\left(k-\tau\right)+u\left(k\right)-\hat{d}\left(k\right)\right)\geq \theta$$

$$\Pr ob\left(d\left(k\right)-\hat{d}\left(k\right)-\delta\times d\left(k-\tau\right)+\delta\times\hat{d}\left(k-\tau\right)\leq S\left(k\right)+u\left(k\right)-\hat{d}\left(k\right)+\delta\times\hat{d}\left(k-\tau\right)\right)\geq \theta$$

$$\Pr ob\left(\frac{d\left(k\right)-\hat{d}\left(k\right)-\delta\times\left(d\left(k-\tau\right)-\hat{d}\left(k-\tau\right)\right)}{V_d\times V_{d,k-\tau}}\leq\frac{S\left(k\right)+u\left(k\right)-\hat{d}\left(k\right)+\delta\times\hat{d}\left(k-\tau\right)}{V_d\times V_{d,k-\tau}}\right)\geq \theta$$

$$\Pr ob\left(\frac{1}{V_{d,k-\tau}}\times\frac{d\left(k\right)-\hat{d}\left(k\right)}{V_d}-\frac{\delta}{V_d}\times\frac{d\left(k-\tau\right)-\hat{d}\left(k-\tau\right)}{V_{d,k-\tau}}\leq\frac{S\left(k\right)+u\left(k\right)-\hat{d}\left(k\right)+\delta\times\hat{d}\left(k-\tau\right)}{V_d\times V_{d,k-\tau}}\right)\geq \theta$$

Avec $\hat{d}(k)$: the demand mean at période *k*

$\hat{d}(k-\tau)$: the demand mean at période (k-τ)

$Var(d(k)) = V_{d,k} > 0$ Variance of demand *d* at period *k*

$Var(d(k-\tau)) = V_{d,k-\tau} > 0$ Variance of demand *d* at period (k-τ)

Note that $X = \left(\frac{d(k)-\hat{d}(k)}{V_{d,k}}\right)$: is a Gaussian random variable with an identical distribution as *d(k)*.

and $Y = \left(\frac{d(k-\tau)-\hat{d}(k-\tau)}{V_{d,k-\tau}}\right)$: is a Gaussian random variable with an identical distribution as $d(k-\tau)$

This formulation is equivalent to $\Pr ob\left(A\times X+B\times Y\leq C\right)\geq\theta$ with $A=\frac{1}{V_{d,k-\tau}}$ et $B=-\frac{\delta}{V_d}$

$X' = A\times X$ is a Gaussian random variable with an identical distribution as $f_{X'} = \frac{1}{A}\times f\left(\frac{y}{A}\right)$, with mean $A\times\hat{d}(k)=\frac{1}{V_{d,k-\tau}}\times\hat{d}(k)$ and variance $A^2\times V_{d,k} = \left(\frac{1}{V_{d,k-\tau}}\right)^2\times V_{d,k}\geq 0$

And $Y' = B\times Y$ is a Gaussian random variable with an identical distribution as $f_{Y'} = -\frac{1}{B}\times f\left(\frac{y}{B}\right)$, with mean $B\times\hat{d}(k-\tau) = -\frac{\delta}{V_d}\times\hat{d}(k-\tau)$ and variance $B^2\times V_{d,k-\tau} = \left(-\frac{\delta}{V_d}\right)^2\times V_{d,k-\tau}\geq 0$

Thus $T' = X' + Y'$ is a Gaussian random variable with an identical distribution as $h = f_{X'} * f_{Y'}$, with mean $A\times\hat{d}(k)+B\times\hat{d}(k-\tau)$ and variance $A^2\times V_{d,k}+B^2\times V_{d,k-\tau}\geq 0$

ϕ **:** is a probability distribution function of T'

$$\phi\left(\frac{S(k)+u(k)-\hat{d}(k)+\delta\times\hat{d}(k-\tau)}{V_{d,k}\times V_{d,k-\tau}}\right)\geq\theta \tag{11}$$

Since $\lim_{-\infty}\phi\rightarrow 0$ and $\lim_{-\infty}\phi\rightarrow 1$ we conclude that ϕ is strictly increasing. We note that ϕ is indefinitely differentiable, so we conclude that φ is invertible.

$$\frac{S(k)+u(k)-\hat{d}(k)+\delta\times\hat{d}(k-\tau)}{V_{d,k}\times V_{d,k-\tau}}\geq\phi^{-1}(\theta) \tag{12}$$

$$u(k)\geq(V_{d,k}\times V_{d,k-\tau})\times\phi^{-1}(\theta)+\hat{d}(k)-\delta\times\hat{d}(k-\tau)-S(k)$$

Thus

$$\Pr ob(s(k+1)\geq 0)\geq\theta \;\Rightarrow\; \left(U(k)\geq(V_{d,k}\times V_{d,k-\tau})\times\phi^{-1}(\theta)+\hat{d}(k)-\delta\times\hat{d}(k-\tau)-S(k)\right) \qquad k=0,1,....,H-1$$

MAINTENANCE POLICY

Our maintenance policy adopted in the problem is a periodic preventive maintenance policy with minimal repair. More precisely, the machine will operate over a given horizon *H*. the maintenance policy adopted is as follows: the *H* production periods is divided equally into *N* parts of duration *T*. Perfect preventive maintenance actions are is performed periodically at times *j.T*, *j*=0,1,…,*N*, and *N.T*=*H* following which the unit is as good as new. Whenever a failure occurs between preventive maintenance actions, the system undergoes a minimal repair. It is assumed that the repair and replacement times are negligible.

It’s obvious that the maintenance policy is tightly related to the system degradation. That is why, the maintenance plan should be optimized considering the production plan previously established for the *H* periods of the planning horizon, in order to take into account the influence of the production rate on the failure rate $\lambda(t)$.

The analytic expression of the total expected maintenance cost is as follows, with *T*=*H/N*, $N\in\{1,2,3\ldots\}$

$$\varphi(N,U)=(N-1)\times M_p+M_c\times\chi(U,N)$$

Each period *k* of the horizon $H.\Delta t$ is characterized by its own production rate $u(k)$ established from the production plan. The failure rate evolves in each interval according to the production rate adopted in that interval. It also depends on the failure rate cumulated at the end of the previous period. The degradation in the end of the period is then accounted for. In fact, the failure rate in the interval *k* is expressed as follows:

$$\lambda_k(t)=\lambda_{k-1}(\Delta t)+\frac{u(k)}{U_{\max}}\cdot\lambda_n(t) \tag{13}$$

With $\lambda_0(0)=\lambda_0$ and $\Delta\lambda_k(t)=\frac{u(k)}{U_{\max}}\cdot\lambda_n(t)$

$\lambda_n(t)$ is the nominal failure rate corresponding to the maximal production rate.

We recall that assumed that machine degradation varies linearly according to the production rate.

We can write the failure rate function as expressed in the following:

$$\lambda_k(t)=\lambda_0+\sum_{l=1}^{k-1}\frac{u(l)}{U_{\max}}\lambda_n(\Delta t)+\frac{u(k)}{U_{\max}}\lambda_n(t) \text{ with } t\in[0,\Delta t] \tag{14}$$

We noticed that the maintenance policy is tightly related to the system degradation. That is why we adopted the production level in order to take into account the influence of the production rate on the failure rate $\lambda(t)$. Letting $L(T)=\int_0^T \lambda_s(t)$ denotes the average number of failures incurred over the interval [0, T], the average failure number over the horizon H is:

$$\chi(u,T)=\sum_{j=0}^{N-1}\left(\sum_{i=In\left(j\times\frac{T}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{T}{\Delta t}\right)}\int_0^{\Delta t}\lambda_i(t)\,dt+\int_0^{(j+1)\times T-In\left(\frac{(j+1)\times T}{\Delta t}\right)\times\Delta t}\lambda_{In\left(\frac{(j+1)\times T}{\Delta t}\right)+1}(t)\,dt+\int_{(j+1)\times T}^{\left(In\left(\frac{(j+1)\times T}{\Delta t}\right)+1\right)\times\Delta t}\frac{u\left(In\left(\frac{(j+1)\times T}{\Delta t}\right)+1\right)}{U_{max}}\times\lambda_0(t)\,dt\right) \quad (15)$$

with $T=H/N$

$$\chi(U,N)=\sum_{j=0}^{N-1}\left(\begin{array}{l}\left(In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)-In\left(j\times\frac{H}{N\cdot\Delta t}\right)\right)\times\Delta t\times\lambda_0(t_0)+\\ \frac{\lambda_0(\Delta t)\times\Delta t}{U_{max}}\times\sum_{i=In\left(j\times\frac{T}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)}\sum_{l=1}^{i-1}u(l)\,dt+\frac{1}{U_{max}}\cdot\sum_{i=In\left(j\times\frac{H}{N\cdot\Delta t}\right)+1}^{In\left((j+1)\times\frac{H}{N\cdot\Delta t}\right)}\int_0^{\Delta t}u(i)\cdot\lambda_n(t)\,dt+\\ \sum_{l=1}^{In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)}\frac{u(l)}{U_{max}}\cdot\lambda_n(\Delta t)\cdot\left((j+1)\times\frac{H}{N}-In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)\times\Delta t\right)\\ +\int_0^{(j+1)\times\frac{H}{N}-In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)\times\Delta t}\frac{u\left(In\left(\frac{(j+1)\times H}{N\cdot\Delta t}\right)+1\right)}{U_{max}}\cdot\lambda_n(t)\,dt\\ +\frac{u\left(In\left(\frac{(j+1)\cdot H}{N\cdot\Delta t}\right)+1\right)}{U_{max}}\times\int_{In\left((j+1)\cdot\frac{H}{N\cdot\Delta t}\right)\times\Delta t}^{(j+1)\frac{H}{N}}\lambda_n(t)\,dt\end{array}\right) \quad (16)$$

NUMERICAL EXAMPLE

In order to illustrate the model developed previously, we consider a company represented by machine M1 which has to satisfy a stochastic demand assumed Gaussian over a finite horizon $H=60$. The machine M1 has a degradation law characterized by a Weibull distribution.

(iii) The following data are used for the other parameters: Cpr=3mu, Cs=2mu, U_{max}=500, $\delta \in$ [0,0.5] , τ=1, service level θ=0.95, initial inventory S_0=0. The variance $V_{d_k}=1.41$. The mean demand is presented in the table 1.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
8	8	9	8	8	8	7	6	4	5	7	8
10	8	9	5	6	6	9	9	7	8	7	10
7	5	4	6	7	8	8	8	9	8	8	6
6	9	9	7	6	4	5	7	8	10	7	8
9	10	2	6	8	9	9	9	10	2	3	5

Table 1: Mean demand

Applying the numerical procedure we obtained the optimal production plan and the optimal maintenance period.
The economical production plan corresponding to the previous demand for δ=0.1 and δ=0.3**,** is as following tables.
δ=0.1

2	**10**	**10**	**10**	**10**	**7**	**4**	**3**	**2**	**5**
10	**9**	**10**	**6**	**10**	**2**	**3**	**5**	**10**	**10**
4	**10**	**4**	**10**	**5**	**2**	**2**	**9**	**8**	**9**
7	**7**	**10**	**5**	**7**	**2**	**5**	**10**	**10**	**4**
4	**2**	**5**	**10**	**9**	**10**	**3**	**10**	**10**	**10**
2	**2**	**8**	**10**	**8**	**8**	**10**	**2**	**2**	**2**

δ=0.3

2	**10**	**10**	**4**	**6**	**6**	**3**	**2**	**2**	**4**
9	**7**	**10**	**2**	**9**	**2**	**2**	**2**	**10**	**8**
2	**7**	**2**	**10**	**2**	**2**	**2**	**6**	**6**	**7**
5	**5**	**9**	**3**	**6**	**2**	**3**	**10**	**8**	**2**
2	**2**	**3**	**9**	**7**	**10**	**2**	**6**	**8**	**9**
2	**2**	**4**	**7**	**6**	**6**	**9**	**2**	**2**	**2**

For the maintenance policy, the scale and shape parameters of the Weibull distribution are respectively β=100 and α=2, while M_c =*3000* mu, M_p=*500* mu, and Δt=1.

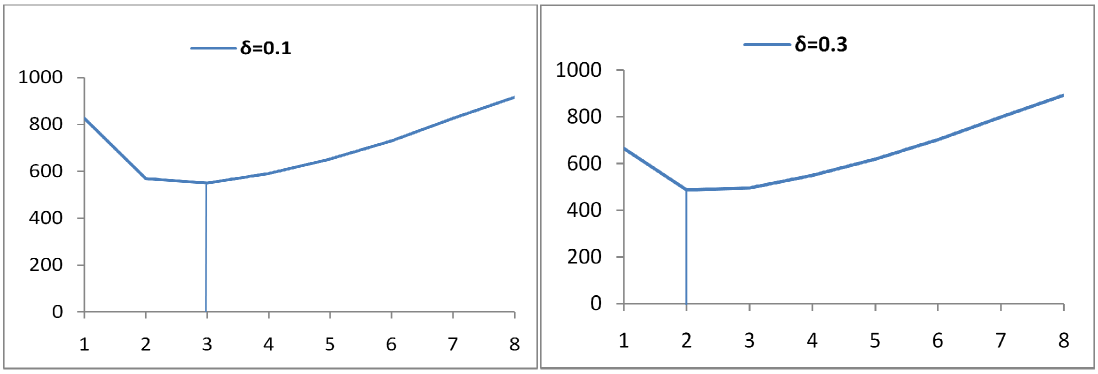

Fig.2. Curves of total cost of maintenance according to *N*

According to previous figures, we notice the influence of product returns in terms of optimal preventive maintenance period. Figure 3 show the total cost of maintenance, $\varphi(N)$**,** according to *N*. The optimal number of partitions *N* * Maintenance is:
For δ =0.1: N^*=3, φ^*=550.09*mu* and the optimal period of maintenance $T^*=H/N^*=20\Delta t$
For δ =0.3: N^*=2, φ^*= 487.94mu and the optimal period of maintenance $T^*=H/N^*=30\Delta t$
In the same way as the previous comment about the production plan, the period of maintenance has become increasingly important when $\boldsymbol{\delta}$ value increases. This is logical δ grows, so that *u* is decreasing, thus the preventive maintenance actions are less.

CONCLUSION

In this paper, we studied a problem of an integrated production/maintenance policy for a manufacturing system facing a random demand on a finite horizon and given a certain service level. Points of view reliability, a minimal repair is practiced at every failure. In order to reduce the failure frequency, preventive maintenance actions is scheduled according to the production rate. The key of this study is to consider that the failure rate increases with time and according to the production rate.

Firstly, we formulated and solved a linear-quadratic stochastic production problem with take into account the return of product by customer (withdrawal right). Using the HMMS model, the plan minimizes the production and the inventory cost with a variable production rate.

Secondly, we introduced a preventive maintenance strategy taking into consideration the influence of the production plan on the manufacturing system deterioration. We used the economical production plan in the maintenance total cost. The objectives are finding out the partition number of the production horizon H after which a preventive maintenance is required.

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